Angle Estimation using Super Resolution and Blocking Matrix in Stepped Multiple Frequency Complementary Phase Code Radar

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Abstract—In this paper, two angle estimation methods for applying the stepped multiple frequency CPC radar are considered. One is 2-D Super Resolution (2-D SR) (Method I) and the other is the combination of 1-D SR and Blocking Matrix method (BM) plus monopulse angle estimation (Method II). From the simulation results, the range and angle are obtained by both methods even in the situation where the targets have same velocity and located on the very close range and angle each other. RMSEs of Method I for both range and angle are smaller than those of Method II. On the other hand, both random and bias errors of Method II are smaller than those of Method I in the experiments. The results indicated that Method II has a tolerance to the calibration errors that exists in the actual measurements.

Keywords—Radar,Stepped multiple frequency CPC, Array antenna, Super-resolution method

I. INTRODUCTION

In recent years, radar is expected to be utilized as a part of Intelligent Transport System (ITS). Automotive radar has been generally used in applications such as Autonomous Cruise Control (ACC), Collision Avoidance etc. In these applications, radar is required to detect targets in long range and achieve a high range resolution. Radar is also expected to estimate the angle of the target to know the target positions. Pulse compression radar [1] [2] is a common technique to realize a high range resolution equivalent to the transmission bandwidth. Meanwhile pulse compression generally needs the wide band receiver to accomplish the high range resolution, which decreases S/N ratio and requires a heavy computational load.

From the background described above, we have proposed Stepped multiple frequency complementary phase code (CPC) modulation [3] [4]. The unique radar modulation/demodulation method can achieve a high range resolution and a long-range detection performance by a narrow receiver bandwidth compared to total transmitting bandwidth. That is why this method has long range detection performance [5]. This method also made it possible to obtain the extremely low range sidelobe by the short code length in CPC pulse compression. Authors have developed and are developing 60GHz, 76GHz, and 79GHz millimeter wave radars using stepped multiple frequency CPC, respectively [6]. These millimeter wave radars meet the specified low-power radio station standard of the millimeter wave in Japan.

In the application of automotive radar, radar is also expected to estimate the angle of the target to know whether it is located in the same lane or the next lane. The number of antennas in Takayuki Inaba The University of Electro-Communications (UEC), Tokyo 1828585, Japan Email: inaba@ee.uec.ac.jp

such a system is hoped to be small from the point of view of the cost and space. In addition, it is hoped to be robust to mutual coupling between elements and to have a tolerance to the calibration errors that exists in the actual measurements [7]. That is why the angle estimation of the millimeter wave radar developed by our research group adopted mono-pulse angle estimation. The received signal is filtered by the velocity and range estimation before the angle estimation. In most cases, the signal in a velocity and range bin at the input of angle estimation involves the information of only a single target even in multiple target situations [8]. Therefore the mono-pulse angle estimation can work well even in the situations. In this paper, we consider more severe situations where the targets have same velocity and are located in very close range each other which is comparable to the range resolution of the radar. For these situations, we have proposed a way to apply 2-D MUSIC method to the stepped multiple frequency CPC to estimate range and angle of the targets (Method I) [9] [10] [11]. In this paper, we consider the angle estimation method using combination of Super-Resolution (SR) and Blocking Matrix method (BM) plus mono-pulse angle estimation. We will show not only the simulation results but also the experimental results conducted in an anechoic chamber.

II. STEPPED MULTIPLE FREQUENCY CPC RADAR

Stepped multiple frequency CPC modulation is a hybrid method of synthetic bandwidth and CPC pulse compression. As shown in Fig.1 (a), the pair of CPC pulses (Code 1 and 2) which satisfy the complementary condition each other are coded on the same carrier frequency. The transmitted carrier frequencies are changed N times step-like in a sequence. The same sequences are repeated M times in a CPI.

A. Block diagram of signal processing of Stepped multiple frequency CPC radar

Fig.1 (b) shows the block diagram of the signal processing of Stepped multiple frequency CPC. After Pulse Compression process, Doppler frequencies are estimated by FFT in pulse hit direction on the same range bin of Pulse Compression output, which corresponds to Pulse Doppler Filter (PDF) process of Pulse compression radar. The CPC pulse compression that is the combination of pulse compression, compensation of Doppler phase shift, and Adding (ADD) process provides the range gate for the subsequent Synthetic Wideband Waveform (SWW) with extremely low range side-lobe. Finally, SWW produces Range-Doppler map with a high range resolution (b)



Fig.1 Stepped multiple frequency CPC modulation / demodulation method ((a) Transmitting sequence of stepped multiple frequency CPC, (b) Schematic diagram of the signal processing)

equivalent to the transmission bandwidth by a narrow band receiver compared with the transmission bandwidth.

B. Signal processing of Stepped multiple frequency CPC radar

The received signal of Stepped multiple frequency CPC is expressed by equation (1) [3].

$$R(t) = \exp\left\{j\frac{4\pi V_{tg}}{c}\left(fc + n\Delta f\right)\left(Tpri\cdot\left(2n + code\right) + Tpri\cdot2\cdot N\cdot m + \frac{2R_{tg}}{c}\right) - j\frac{4\pi R_{tg}}{c}\left(fc + n\Delta f\right) + j\frac{2\pi d}{\lambda}\sin\left(\theta_{tg}\right)\cdot l\right\}\cdot\exp\left(j\varphi_{code}\left[t - \frac{2R_{tg}}{c}\right]\right)$$
(1)

where V_{tg} , R_{tg} , and θ_{tg} are velocity, range, and angle of the target, respectively. *d* and *l* indicate distance of receivers and receivers index. Pulse compression is done for each PRI. Since the matched filter is a linear time invariant system, its output can be described mathematically by the convolution between the received signal R(t) and reference signal Ref(t). The output of the process is denoted by the function of sequence number *m* and frequency step *n*, and complementary code number *code* (*code*=1,2), and range bin *s*.

$$PC[code, n, m, s] = \mathfrak{I}^{-1}(\mathfrak{I}(R) \cdot \mathfrak{I}(Ref))$$
(2)

PDF is also performed in sequence direction (*m* direction in Fig.1 (a)) on the same range bin, which is described by (3). By PDF process, the sequence number *m* is converted into Doppler bin of k.

$$PD[code, n, k, s] = \sum_{m=0}^{M-1} PC[code, n, m, s] \cdot \exp\left(-2\pi j \cdot \left(\frac{m}{M}k\right)\right)$$
(3)

CPC pulse compression generally suffers from the phase shift due to the Doppler frequency. Stepped multiple frequency CPC has the compensation process for the phase shift.

$$PHC[code, n, k, s] = PD \cdot \exp\left\{2\pi j \frac{m}{2MN \cdot PRI} \left(s + PRI \cdot \left(2n + code\right)\right)\right\}$$
(4)

After the phase compensation process, CPC pulse compression is completed by ADD process where the output of code 1 is added by that of code 2.

$$ADD[n,k,s] = \sum_{code=0}^{1} PHC[code,n,k,s]$$
(5)

As a result, the range profile has extremely low range sidelobe. Finally, SWW, which is realized by FFT in frequency direction (n direction) produces a high range resolution equivalent to the transmission bandwidth by a narrow band receiver compared with the transmission bandwidth.

$$SWW[k,s'] = \sum_{n=0}^{N-1} ADD[n,k,s] \exp\left(j \cdot \left(\frac{4\pi \left(dR \cdot xr + s \cdot \Delta R\right)}{c}n \cdot \Delta f\right)\right)$$
$$s' = N \cdot s + xr \tag{6}$$

where xr is range sample after SWW process.

III. PROPOSED ANGULLAR ESTIMATION METHOD OF STEPPED MULTIPLE FREQUENCY CPC RADAR

The radar system developed by our research group has 4 receiving antennas. We have proposed 2-D SR (2-D MUSIC) method for applying to the output of ADD process to estimate the range and angle in the high resolution [9] [10]. In this paper, the method is also briefly reviewed.

A. 2-D Super Resolution

In this section, we describe the range/angle estimation method based on 2-D MUSIC algorithm. After ADD process, the output at a range bin of s and at a Doppler bin k, where the target exist is described as

$$4DD_{k,s}\left[n,l\right] = \sum_{lg}^{Tg} \exp\left[j\left(-\frac{4\pi R_{lg}}{c}\right)(fc + n\Delta f) + j\left(2\pi \frac{d}{\lambda}\sin(\theta_{lg}) \cdot l\right)\right]$$
(7)



Fig.2 Block diagram of angle estimation in stepped multiple frequency CPC using 2-D SR (Method I) and 1-D SR + BM + Monopulse angle estimation (Method II).

In equation (7), λ is the wavelength of transmitted signals. The number of antenna is *L*. Thus after ADD process, we obtain data matrix $\mathbf{X} \in \mathbb{C}^{N \times L}$ for each range and Doppler bin (*s* and *k*). The correlation matrix $\mathbf{R} \in \mathbb{C}^{Np \times Lp}$ (Np < N, Lp < L) is obtained by ensemble-averaging sub correlation matrix [9] [10]. The MUSIC spectrum is computed by performing an eigen-analysis on **R**. The noise subspace is given by $\mathbf{E}_N = [\mathbf{e}_{Tg+1}, \mathbf{e}_{Tg+2}, \dots \mathbf{e}_{NqNp}]$, (8)

The *i* th element of steering vector $\mathbf{a}(R,\theta)$ to search the target range/angle is

$$a_{i}(R,\theta) \equiv \exp(j2\pi(\frac{-2\Delta f}{c}R)(\frac{i-\operatorname{mod}(i,Np)}{Np}) + \frac{d}{\lambda}\sin(\theta)\cdot\operatorname{mod}(i,Np))$$

$$\in C^{(Np\cdot Lq)\times 1} , \qquad (9)$$

where mod(x, y) is an operator calculating a remainder of x/y. The 2D MUSIC spectrum is also given by the following equation

$$MUSIC(R,\theta) = \frac{a^{H}(R,\theta)a(R,\theta)}{a^{H}(R,\theta)E_{N}E_{N}^{H}a(R,\theta)}$$
(10)

B. 1-D Super Resolution plus Blocking Matrix

In this section, angle estimation method that is mono-pulse angle using combination of 1-D SR and Blocking Matrix method (BM) is described.

Method II of Fig.2 shows the angle estimation using the combination of 1-D SR and Blocking Matrix method (BM) plus monopulse angle estimation after ADD process of the Stepped multiple frequency CPC radar. The steering vector of 1-D SR for range estimation is described by a(r). MUSIC spectrum is given by the equation (11) [12].

$$\mathbf{P}_{MU}(r) = \frac{a^{H}(r)a(r)}{a^{H}(r)E_{N}E_{N}^{H}a(r)}$$

$$\left(a(r) = \exp\left[-j\left(\frac{4\pi r}{c}\Delta f\right)n\right], \quad E_{N} = \left[e_{Tg+1}, \dots, e_{Np}\right]\right)$$
Using the constant halo exists the balance is to the main

Here, $\mathbf{e}_{T_{g+1},...,} \mathbf{e}_{N_p}$ are the eigenvectors belonging to the noise subspace. At first, we estimate the ranges of targets (r₁, r₂) using 1D-SR. 1-D SR is expected to separate the targets even

in the situation that targets located at very close range each other, which cannot separate by normal SWW (see Fig.3 (a) and (b)). The input y_l of the monopulse angle estimation for the targets 1 and 2 using 1-D SR method is generally considered to be recovered by signal copy (SC) that is equivalent to SWW for a single range bin. In this paper, before SC we create a projection matrix (PM) P_2 and P_1 to suppress the signal component corresponding to r_2 and r_l obtained by 1-D SR to estimate the angle for target 1 and 2 (see Fig.3 (b)), respectively. The output data vector of ADD process of each antenna $x_l \in C^{N \times 1}$ is multiplied by P from left (BM). The input of mono-pulse angle estimation is produced by inner product of $a(r_1)$ and P_2x_l . The same process is performed for target 2 (see Fig. 3 (c) and (d)).

$$y_{l} = a^{H}(r_{1})P_{2}x_{l} \qquad \left(P_{2} = I - \frac{a(r_{2})a(r_{2})^{H}}{a(r_{2})^{H}a(r_{2})}\right) \quad \text{(for target 1)}$$
$$y_{l} = a^{H}(r_{2})P_{1}x_{l} \qquad \left(P_{1} = I - \frac{a(r_{1})a(r_{1})^{H}}{a(r_{1})^{H}a(r_{l})}\right) \quad \text{(for target 2) (12)}$$

Then we calculate each target angle using the phase difference of y by conventional mono-pulse angle estimation [13].



Fig.3 Output of each process ((a) SWW result of Stepped multiple frequency CPC in Fig.1 (b), (b) 1D-SR(MUSIC) spectrum using ADD output of Method II in Fig 2, (c) SWW result after BM for target 1 of Method II, (d) SWW result after BM for target 2 of Method II).

IV. SIMULATION RESULTS

The simulation was conducted to verify the basic performance of Method II by employing the following parameters of the stepped multiple frequency CPC radar described in table 1. The targets are located at the range of 3.068m and 3.239m. The range difference 17cm corresponds to the 1/2 of range resolution of transmission bandwidth of 430MHz. As shown in Fig.3 (a), we cannot separate the targets whose range difference is smaller than the range resolution by the Fourier Transform based processing (SWW).

Fig. 4 shows a result of 2-D SR, where the input S/N for 2-D SR is set to be 20dB. From Fig.4, the ranges of two targets are obtained to be 3.070m and 3.243m, respectively. The angles of the two targets are also obtained to be -1.1deg and 1.0deg, respectively. From Fig.4, 2-D SR is seemed to work well in the target situation and S/N condition.

Fig.5 (a) shows range estimation result of 1-D SR in Method II for the same data of Fig.4. From 1-D SR, we can obtain the ranges 3.064m and 3.240m for target 1 and 2, respectively. By using the range estimation results of 1-D SR, PMs are generated for individual ranges. As shown in Chapter III, we can obtain *y* for angle estimation.

Fig.5 (b) shows the results of the mono-pulse angle estimation for target 1 and 2. From Fig.5 (b), the angles for target 1 and 2 are obtained to be -0.8 deg and 0.9deg, respectively.

Fig. 6 (a) shows the dependence of the RMSE of range estimation on S/N at ADD process. From Fig.6, RMSE of 2-D SR is smaller than that of 1-D SR in Method II. Fig. 6 (b) shows the dependence of the RMSE of angle estimation on S/N. As in the case with range estimation, RMSE of 2D-SR is smaller than that of mono-pulse angle estimation in Method II.

Table1. Radar Parameters of Stepped multiple frequency CPC in the simulation.

Radar Parameters	Specifications
Transmit frequency	60.32-60.67GHz
Pulse bandwidth	80MHz
Code length	16
Number of codes consisting sequence set :CODE	2
Pulse Repetition Interval (PRI)	3.5µsec
Number of Sequence Repetition: M	512
Frequency step width	50MHz
Frequency step number: N	8
Transmission bandwidth	430MHz
Observation time (CPI)	29msec
A/D sampling frequency	160MHz
Detectable Velocity	\pm 79km/h
Velocity Resolution	0.31km/h
Range Resolution	0.34m

Table2. Target parameters in the simulation.

	Range	Angle	Velocity
Target 1	3.068m	-1.0 deg	4km/h
Target 2	3.239m	1.0deg	4km/h



Fig.4 Range and angle estimation result of 2-D SR (Method I). The input S/N ratio is 20dB.



Fig.5 Range and angle estimation result of Method II. The input S/N ratio is 20dB.



Fig.6 Simulation results for the dependence of RMSE on input S/N ratio ((a) RMSE of range, (b) RMSE of angle).

V. EXPERIMENTAL RESULTS

The experimental verification was also conducted in an anechoic chamber using same radar parameters as the simulation described in section IV. The stepped multiple frequency CPC radar used in the experiment has an array antenna composed of 4 receiving element. Beam width of the four elements by DBF is 13deg. We use two corner reflectors of 10 dBsm. The target conditions are shown in Table 3. As shown in Fig. 7, the two corner reflectors are moved at the same velocity of 4km/h with keeping the same range difference of 17cm by using an actuator. The experiments described above are conducted 32 times for the same situation.

Fig.8 shows a result (3rd data) of 2-D SR of Method I, where the input S/N for 2-D SR is about 36dB in the situation. From Fig.8, the ranges of two targets are obtained to be 19.703m and 19.497m, respectively. The angles of the two targets are also obtained to be -0.1deg and 1.1deg, respectively. From Fig.8, 2-D SR is seemed to work in this case, although the bias is found in the angular estimation. Fig. 9 shows a result (9th data) of 2-D SR of Method I. It is difficult to identify the two peaks in Fig.9, since the MUSIC spectrum has only a single peak. The MUSIC spectrum has a single peak in 11 out of 32 measurements. In other words, we cannot separate the two targets by 2-D SR.

Fig.10 (a) shows the range estimation result of 1-D SR in Method II for the data of Fig.8 (3rd data). From 1-D SR, we can obtain the ranges 19.684m and 19.488m for target 1 and 2, respectively. The angles of the two targets are also obtained to be -0.9deg and 1.2deg, respectively. Fig. 11 (a) shows the range estimation result of 1-D SR in Method II for the data of Fig.7 (9th data). From 1-D SR, we can obtain the ranges 19.200m and 19.020m for target 1 and 2, respectively. The angles of the two targets are also obtained to be -0.9deg and 1.2deg, respectively, although 2-D SR cannot separate the two targets (Fig. 11 (b)).

The experimental results are summarized in Table 4. Unlike the simulation results, both random and bias errors of angle by Method II are smaller than those of Method I. By Method II, two targets could be identified in all cases. From the initial experimental result, the proposed method could obtain the target angle by mono-pulse angle estimation for the targets which have same velocity and are located on very close range, which is equivalent to the 1/2 range resolution.



Fig.7 Experiment in an anechoic chamber using stepped multiple frequency CPC radar.

Table3 Target parameters of the experiment in the anechoic chamber

	Range	Angle	Velocity	
Target 1	18.67-20.27m	-1.0 deg	4km/h	-
Target 2	18.50-20.10m	1.0deg	4km/h	_



Fig.8 Range and angle estimation result for 3rd data by 2-D SR (Method I).



Fig.9 Range and angle estimation result for 9th data by 2-D SR (Method I).



Fig.10 Range and angle estimation result for 3rd data by Method II ((a) Range estimation by 1-D SR, (b) Angle estimation by mono-pulse angle estimation).



Fig.11 Range and angle estimation result for 9th data by Method II ((a) Range estimation by 1-D SR, (b) Angle estimation by mono-pulse angle estimation)

Table 4. Experimental results by two methods (Method I and II)

	Method I	Method II (1 D SR+RM+Mono pulse)
	(2-D SK)	(1-D SK + DWI + Wiolio-puise)
Mean of range difference between targets	0.186 m	0.188 m
Standard deviation of range difference	0.023	0.027
	Method I	Method II
	(2-D SR)	(1-D SR+BM+Mono-pulse)
Mean of estimated angle between targets	1.18 deg	2.00 deg
Standard deviation of estimated angle	0.83	0.20

VI. DISCUSSIONS AND CONCLUSTIONS

In the application of automotive radar, radar is expected to obtain the angle of the target. It is also hoped to be robust to mutual coupling between elements and to have a tolerance to the calibration errors that exists in the actual measurements. That is why the angle estimation of the millimeter wave radar developed by our research group adopted mono-pulse angle estimation. The mono-pulse angle estimation can work well even in the multiple target situations, since the same range and Doppler bin at the input of angle estimation has only single target information after being filtered by velocity and range (PDF and SWW).

In this paper, we consider more severe situations where the targets has same velocity and are located in very close range each other which is comparable to the range resolution of the radar. We considered two angle estimation methods for applying stepped multiple frequency CPC radar. One is 2-D SR and the other is the combination of 1-D SR and BM plus mono-pulse angle estimation for the situation where the targets has same velocity and are located on very close range which is comparable to the range resolution. From the simulation results, the bias error of both proposed method is less than 0.02m and 0.5 deg from the true value, when the input S/N is larger than 15dB. RMSEs of Method I for both

range and angle are smaller than those of Method II.

In the experiments, the random and bias errors of range are similar tendency by two methods. On the other hand, both random and bias errors of angle by Method II are smaller than those of Method I in the experiments. By Method II, two targets could be identified in all cases, although we could not separate the two targets by 2-D SR. In the measurement, the amplitude gain and phase offset for each frequency step and for each antenna are calibrated in the signal processing. The element pattern is also considered. However, it is difficult to calibrate perfectly all components, which is especially difficult for antenna elements. Thus the calibration errors are remained in the actual measurements. It is also indicated that Method II has a tolerance to the calibration errors that exists especially in antenna elements in the actual measurements. In other words, Method II is possible to be a good candidate of the signal processing for the next generation automotive radar.

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REFERENCES

- [1] M. I.Skolnik, Introduction to Radar Systems, McGraw-Hill, New York,
- pp.81-92, 1962 T. Kishigami, T. Morita, H. Yomo, M. Yasugi, and Y. Nakagawa, Advanced wide field of view millimeter-wave radar using orthogonal Peder Conference 2014. [2] complementary codes, Radar Conference 2014 10.1109/RADAR.2014.6875649, 2014
- [3] M. Watanabe, M. Akita, and T. Inaba, Stepped Multiple Frequency Complementary Phase Code Radar and the Fundamental Experiment, IEEJ Transactions on Electronics, Information and Systems Vol. 135, No.3, pp. 285-291, 2015 (in Japanese).
- [4] R. Craigen, W. Holzmann, and H. Kharaghani. Complex Golay sequences: structure and applications. Discrete Math., 252:73–89, 2002.
- [5] M. Akita, Y. Ota, M. Watanabe, and T. Inaba, Experimental Comparison of Stepped Multiple Frequency CPC with Pulse Compression, 2017 IEEE MTT-S International Conference on Microwaves for Intelligent Mobility (ICMIM), DOI: 10.1109/ICMIM.2017.7918870, 2017
- [6] M. Akita, M. Watanabe, T. Inaba, Development of millimeter wave radar using stepped multiple frequency Complementary Phase Code and concept of MIMO configuration, 2017 IEEE Radar Conference (RadarConf), DOI: 10.1109/RADAR.2017.7944184, 2017
- [7] M. Schoor and B. Yang, Subspace based DOA estimation in the presence of correlated signals and model errors, 2009 IEEE International Conference on Acoustics, Speech and Signal Processing, DOI: 10.1109/ICASSP.2009.4960045, 2011
- [8] M. Watanabe, M. Akita, and T. Inaba, Fundamental Experiments for Application to Railway Environment of Stepped Multiple Frequency Complementary Phase Code Radar, IEEJ Transactions on Industry Applications, Vol. 135 (2015) No. 5 P 513-520, 2015 (in Japanese)
- T. Inaba, Super Resolution Range/Angle Estimation using Stepped Multiple Frequency Interrupted CW Radar, IEICE Trans.Commun., vol.J107-B No.260, pp63-68, 2007
- [10]M. Akita. R. Yamashita, M. Watanabe, T. Inaba, Fundamental Experiments on Super Resolution Range/Angle Estimation by Millimeter Wave Radar using Stepped Multiple Frequency Complementary Phase Code Modulation, IEICE Technical Report, Vol.114, No.264, pp.101-106, Melaka, Malaysia, 2014.
- [11]F. Belfiori, W. Rossum, and P. Hoogeboom, Coherent MUSIC technique for range/angle information retrieval, IET Radar, Sonar & Navigation, Vol. 8, No.2, pp. 75-83, 2014
- [12] T. Inaba, Multiple Target Detection for Stepped Multiple Frequency Interrupted CW Radar, IEICE Trans.Commun., vol.J89-B No.3, pp.373-383. 2006
- [13] D. Barton, S. Sherman, Monopulse Radar Theory and Practice 2nd ed., Artech House, 2011