

# Coherently Combining Sparse-Multiband Processing for High Range Resolution by Narrow Band Radars

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**Abstract** Authors have been developing a range estimation method for a high range resolution by coherent signal processing using radar data operated in separated frequency bands. It is expected that this technique can obtain the high resolution avoiding the increase of a hardware load without the degradation of the detectable range, which generally become a problem with the expansion of the transmission bandwidth. In this paper, we explain the receiving signal model before synthetic bandwidth of multiple frequency (MF) radar operated in separated frequency bands. The coherently combining sparse-multiband (CCSM) processing which is iterative signal processing composed by signal extraction, coherently combining, range estimation, and complex amplitude estimation is described. The iterative process is based on RELAX algorithm. We also show the simulation results using 2 and 8 separated narrow frequency bands data where two targets are separated with 0.170m and 0.042m equivalent to the range resolution of total transmission bandwidth, respectively.

**Keyword** Radar, Ultra-wide band, Coherent processing, High range resolution, Synthetic bandwidth radar

## 1. Introduction

In recent years, radar is expected to be utilized as a part of Intelligent Transport System (ITS). Automotive radar has been generally used in applications such as Autonomous Cruise Control (ACC), Collision Avoidance etc. By a revision of a radio law, 76GHz band, which will be expanded to 1GHz bandwidth, and 79GHz band, which will be expanded to 4GHz bandwidth, are combined to be allowed to used 5 GHz bandwidth (Ultra-wideband width) in the near future in Japan. The high range resolution can be achieved by the expanding of the transmission bandwidth to ultra-wide bandwidth, since the range resolution depends on the transmission bandwidth. However, we face the increase of a hardware load and the degradation of the Signal to Noise ratio of the receiver associated with expanding the transmission bandwidth resulting in degradation of detectable range.

Authors have developed millimeter wave radar using stepped multiple frequency Complementary Phase Code (CPC) [1]. It could achieve a high range resolution and a long-range detection performance by a narrow band receiver compared with the transmission bandwidth [2]. We have previously demonstrated that the radar operated in 60GHz/76GHz band with 430MHz bandwidth satisfied the expected performance of the sidelobe and the range

resolution.

Authors are developing a range estimation method with a more high range resolution by the expansion of total frequency bandwidth by coherent signal processing using radar data operated in separated frequency bands. [3] [4] [5]. This method has an advantage of that avoid the deterioration of the detectable range. The method is also expected to enables us to realize to utilize the ultra-wide bandwidth by common hardware systems. In this method, the difference of complex amplitudes between the frequency bands, which is considered to adverse effects on the estimation results, is compensated. In this paper, we explain the receiving signal model before synthetic bandwidth of the multiple frequency (MF) radars [2] [6] [7] operating in separated frequency bands and describe the iterative range estimation method of coherently combining sparse-multiband (CCSM) processing. The method itself can be also applied not only MF radars but to Pulse Compression (PC) radar [8] data after the pulse compression process in frequency domain. We also show the simulation results using 2 and 8 separated frequency bands data where two targets are separated with 0.170m and 0.042m equivalent to the range resolution of total transmission bandwidth, respectively.

## 2. Signal model before synthetic bandwidth of MF radar operated in separated frequency bands

In this section, we formulate the signal model of MF radar operated in the separated frequency band. The synthetic band processing in MF radar can obtain a high range resolution with a narrow bandwidth compared with transmission bandwidth by performing IDFT in the frequency direction on the signal after Pulse Doppler Filter (PDF). For simplicity, we formulate the signal model of the case of continuous wave. The method in this paper can be also applied to MF radar such as stepped multiple frequency CPC [1], stepped multiple frequency interrupted CW [7], and so on. The method also applied to Pulse Compression (PC) radar [8] data after the pulse compression process in frequency domain.

The received RF signal  $r_{k,iF,n}(t)$  from  $k$  th target is denoted by following equation

$$r_{k,iF,n}(t) = \alpha_{k,iF} \cdot \exp\left(-j\left(2\pi(f_{iF} + n\Delta f + f_d(n))t + \frac{4\pi(f_{iF} + n\Delta f)}{c}R_k + \phi_{iF,n}\right)\right) \quad (1)$$

where  $\alpha_{k,iF}$ ,  $f_{iF}$ ,  $\Delta f$ ,  $f_d(n)$ ,  $f_{iF}$ ,  $R_k$ , and  $\phi_{iF,n}$  are complex amplitude of the target, bottom carrier frequency of each band, frequency step width, Doppler frequency of the target, range of the target, and an initial phase, respectively.  $iF$  and  $n$  indicates the number of frequency bands and frequency step. The baseband signal  $x_{k,iF,n}(t)$  after mixing with local signal is denoted by

$$x_{k,iF,n}(t) = \alpha_{k,iF} \cdot \exp\left(-j\left(2\pi f_d(n)t + \frac{4\pi(f_{iF} + n\Delta f)}{c}R_k\right)\right) \quad (2)$$

Thus the signal after PDF in consideration of the difference of Doppler frequency due to the frequency steps is obtained as expressed by equation (3). The signal corresponds to the input data of synthetic band processing.

$$y_{k,iF}(n) = \alpha_{k,iF} \cdot \exp\left(\frac{-j4\pi(f_{iF} + n\Delta f)}{c}R_k\right) \quad (3)$$

Therefore, the observation data after PDF from all targets in observation area  $z_{iF}(n)$  is expressed as a linear summation and Gaussian noise

$$z_{iF}(n) = \sum_{k=0}^{K-1} \alpha_{k,iF} \cdot \exp\left(\frac{-j4\pi(f_{iF} + n\Delta f)}{c}R_k\right) + n(n) \quad (4)$$

The observation data vector  $\mathbf{z}_{iF} \in \mathbf{C}^N$  is also expressed by direction matrix  $\mathbf{A}$  and target amplitude vector  $\mathbf{\alpha}_{iF}$ .

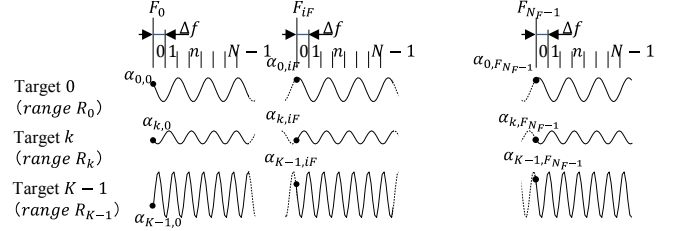


Fig. 1 Illustration of the signal model before Synthetic Bandwidth processing of MF radars operated in separated frequency bands

$$\mathbf{z}_{iF} = \beta_{iF} \mathbf{A}_{iF} \mathbf{\alpha}_{iF} + \mathbf{n} \quad (5)$$

$$\mathbf{A}_{iF} = [\mathbf{a}_{iF}(R_0), \mathbf{a}_{iF}(R_1), \dots, \mathbf{a}_{iF}(R_{K-1})] \quad (6)$$

$$\mathbf{a}_{iF}(R) = \left[ \exp\left(-\frac{j4\pi(f_{iF})R}{c}\right), \exp\left(-\frac{j4\pi(f_{iF} + \Delta f)R}{c}\right), \dots, \exp\left(-\frac{j4\pi(f_{iF} + (n-1)\Delta f)R}{c}\right) \right]^T \quad (7)$$

$$\mathbf{\alpha}_{iF} = [\alpha_{0,iF}, \alpha_{1,iF}, \dots, \alpha_{K-1,iF}]^T \quad (8)$$

where  $\mathbf{a}_{iF}(R)$  is direction vector and  $\beta_{iF}$  is random phase term associated with the small range difference due to the radar locations and snapshots. The signal model before Synthetic Bandwidth processing of MF radars operated in separated frequency bands is also illustrated in Fig.1.

## 3. Signal Processing

Authors are developing a range estimation method with a high-resolution to expand the transmission bandwidth by coherent signal processing using measured radar data operated in separated bands. CCSM processing and the concept have been described in previous literature in Japanese [4] [5]. The modified points from the previous literature are as follows.

- The mode vector  $\mathbf{a}'_{iF}$  is modified to obtain complex amplitude of the targets to connect the target signal between frequency bands without discontinuity in phases. That is described in section 3.1.
- The steering vector to search the likelihood function  $\mathbf{b}_{k,iF}(\mathbf{r})$  described in section 3.2 is modified to be designed in consideration with the distortion of the waveform in subtraction process.
- CCSM is modified to be based on RELAX algorithm, which was based on Cyclic algorithm [9]. That is described in section 3.4.

From Equation (4), range estimation of MF radar is a frequency estimation problem (phase gradient estimation) in frequency direction ( $n$  direction) involved in the observation data. CCSM is composed of signal extraction, coherently combining, range estimation, and complex amplitude estimation, and their iteration. The modified CCSM is described in this chapter.

### 3.1 Signal extraction

The signal components associated targets except for the target for range estimation are subtracted by following subtraction process. The signal associated with only the target for range estimation, (separated signal)  $\mathbf{x}_{k,iF}$  is obtained as described by

$$\mathbf{x}_{k,iF} = \left( \mathbf{I} - \sum_{i,i \neq k}^{K-1} \frac{\mathbf{a}'_{iF}(\hat{R}_i) \mathbf{a}'_{iF}(\hat{R}_i)^H}{\mathbf{a}'_{iF}(\hat{R}_i)^H \mathbf{a}'_{iF}(\hat{R}_i)} \right) \mathbf{z}_{iF} \quad (9)$$

$$\mathbf{a}'_{iF} = \left[ \exp\left(-\frac{j4\pi(f_{iF})R}{c}\right), \exp\left(-\frac{j4\pi(f_{iF} + \Delta f)R}{c}\right), \dots, \exp\left(-\frac{j4\pi(f_{iF} + (n-1)\Delta f)R}{c}\right) \right]^T \quad (10)$$

$$f_{iF} = 0, N\Delta f, \dots, N \cdot iF \cdot \Delta f, \dots, N \cdot (N_F - 1) \cdot \Delta f \quad (11)$$

where  $\mathbf{a}'_{iF}$  denotes the mode vector. The mode vector is modified from literature [4] to obtain complex amplitude of the targets to connect the target signal between frequency bands without discontinuity in phases.  $\hat{R}_i$  denotes estimated range of  $i$  th target. The estimated range for all targets (not true range) is assumed to have been obtained by some way here. This process corresponds to that of CA and RELAX algorithm.

### 3.2 Coherently combining and range estimation

The steering vector  $\mathbf{b}_{k,iF}(\mathbf{r})$  for estimating the range of  $k$  th target is expressed by following equation using estimated target amplitude vector  $\hat{\mathbf{a}}_{iF}$ .

$$\mathbf{b}_{k,iF}(\mathbf{r}) = \mathbf{B} \mathbf{M}_{k,iF} \mathbf{A}'_{iF}(\mathbf{r}) \hat{\mathbf{a}}_{iF} \quad (12)$$

$$\mathbf{B} \mathbf{M}_{k,iF} = \left( \mathbf{I} - \sum_{i,i \neq k}^{K-1} \frac{\mathbf{a}'_{iF}(\hat{R}_i) \mathbf{a}'_{iF}(\hat{R}_i)^H}{\mathbf{a}'_{iF}(\hat{R}_i)^H \mathbf{a}'_{iF}(\hat{R}_i)} \right) \quad (13)$$

$$\mathbf{A}'_{iF}(\mathbf{r}) = [\mathbf{a}'_{iF}(R_0), \mathbf{a}'_{iF}(R_1), \dots, \mathbf{a}'_{iF}(R_{K-1})] \quad (14)$$

$$\mathbf{A}'_{iF}(\mathbf{r})^{<i>} = \begin{cases} \hat{\mathbf{a}}_{iF}(\hat{R}_i) & i \neq k \\ \mathbf{a}'_{iF}(\mathbf{r}) & i = k \end{cases} \quad (15)$$

where  $^{<i>}$  denotes  $i$  th column vector of a matrix. Note that the distortion of the waveform by subtraction process is also considered to make the steering vector. The distortion is not considered in original CA and RELAX algorithm. The range of  $k$  th target is estimated by searching the range where the likelihood function  $LH_k(\mathbf{r})$  described below takes the maximum value. The process has multiple step search, in which coarse search is performed first and then fine search focusing on the result of the coarse search is performed.

$$\hat{R}_k = \operatorname{argmax}_{\mathbf{r}} LH_k(\mathbf{r}) \quad (16)$$

$$LH_k(\mathbf{r}) = \frac{\mathbf{B}_k(\mathbf{r})^T \cdot \mathbf{x} \mathbf{c}_k \mathbf{x} \mathbf{c}_k^T \cdot \mathbf{B}_k(\mathbf{r})}{\mathbf{B}_k(\mathbf{r})^T \cdot \mathbf{B}_k(\mathbf{r})} \quad (17)$$

$$\mathbf{B}_k(\mathbf{r}) = [\mathbf{b}_{k,0}(\mathbf{r}), \mathbf{b}_{k,1}(\mathbf{r}), \dots, \mathbf{b}_{k,N_F-1}(\mathbf{r})] \quad (18)$$

$$\mathbf{x} \mathbf{c}_k = [\mathbf{x}_{k,0}, \mathbf{x}_{k,1}, \dots, \mathbf{x}_{k,N_F-1}] \quad (19)$$

where  $N_F$  is the number of frequency bands.  $\mathbf{B}_k(\mathbf{r})$  and  $\mathbf{x} \mathbf{c}_k$  are generated by stacking  $\mathbf{b}_{k,iF}(\mathbf{r})$  and  $\mathbf{x}_{k,iF}$  for bands. In this process, the coherent vector dimension is expanded to  $N_F$  times, since data vector  $\mathbf{x} \mathbf{c}_k$  is coherently combined.

### 3.3 Complex amplitude estimation

The target amplitude vector is estimated by following equation using estimated direction matrix  $\hat{\mathbf{A}}_{iF}$ , which corresponds to generalized inverse matrix of the estimated direction matrix.

$$\hat{\mathbf{a}}_{iF} = (\hat{\mathbf{A}}_{iF}^H \cdot \hat{\mathbf{A}}_{iF})^{-1} \cdot \hat{\mathbf{A}}_{iF}^H \cdot \mathbf{z}_{iF} \quad (20)$$

### 3.4 CCSM processing algorithm

In CCSM processing, targets range are estimated recursively by signal processing composed by signal extraction, coherently combining, range estimation, and complex amplitude estimation. The iterative process is based on RELAX algorithm [9]. It is noted that the steering vector to search the likelihood function is designed in consideration with the distortion of the waveform by the subtraction process. CCSM processing algorithm based on RELAX algorithm is listed below.

Fig.2 CCSM processing algorithm based on RELAX

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CCSM processing algorithm

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**for**  $K=1, 2, \dots, K_{max}$

**repeat**

**for**  $k=1, 2, \dots, K$

$i = K - k$

$$\mathbf{x}_{i,iF} = \left( \mathbf{I} - \sum_{m,m \neq i}^{K-1} \frac{\mathbf{a}_{iF}(\hat{R}_m) \mathbf{a}_{iF}(\hat{R}_m)^H}{\mathbf{a}_{iF}(\hat{R}_m)^H \mathbf{a}_{iF}(\hat{R}_m)} \right) \mathbf{z}_{iF}$$

$$\hat{R}_i = \operatorname{argmax}_{\mathbf{r}} LH_i(\mathbf{r})$$

$$\hat{\mathbf{a}}_{iF} = (\hat{\mathbf{A}}_{iF}^H \cdot \hat{\mathbf{A}}_{iF})^{-1} \cdot \hat{\mathbf{A}}_{iF}^H \cdot \mathbf{z}_{iF}$$

**end for**

**until (convergence)**

**end for**

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## 4. Results

The simulations (Case I and II) are conducted employing the radar parameters shown in Table 1. Two targets are separated with 0.17m (12.000m, 12.170m) and 0.042m (12.000m, 12.042m) corresponding with the range resolution of total transmission bandwidth in Case I and II, respectively. Signal to Noise (S/N) ratio of observation data is set to be 30 dB. The simulations are conducted 50 times in each case.

### 4.1 Case I

Fig. 3 shows the range estimation result vs. number of iteration in depicted one case in Case I. From Fig.3, it is found that the estimated ranges of targets converge to the true value (indicated by dashed line) and the likelihood function converges to 1. The range estimation results (12.001, 12.171) are obtained from the convergence values of target ranges. Fig. 4 shows the likelihood functions for targets 1 and 2 at the iteration number of 1000 of the final phase of CCSM processing ( $K=K_{\max}=2$ ) as indicated by arrows in Fig.3. The beam width of the likelihood functions are 0.17m that corresponds to the bandwidth of 0.98GHz. That indicated that the coherently combining in CCSM successfully works. Fig. 5 shows the estimation result of  $\hat{\mathbf{a}}_{IF}$  vs. number of iteration of the final phase of CCSM processing. The dashed lines indicate the true value to be set in the simulation for each target for each frequency band. From Fig.5, it is also found that the estimated  $\hat{\mathbf{a}}_{IF}$  converge to the true complex amplitude. Fig. 6 shows the simulation results of 50 trials ((a) range estimation results vs. number of iteration, (b) histogram of range estimation results). The standard deviations of range estimations are 0.003 for both targets.

Table 1. Radar parameters in the simulation

Radar Parameters	Specifications	
	Case I	Case II
Transmit frequency	79-83GHz	79-83GHz
Total transmission bandwidth	0.976GHz	3.504GHz
Frequency step width	13.4MHz	13.4MHz
Step number in sub-band	32	32
Sub-band bandwidth	438MHz	438MHz
Number of Subband	2	8
Frequency step number	64	256

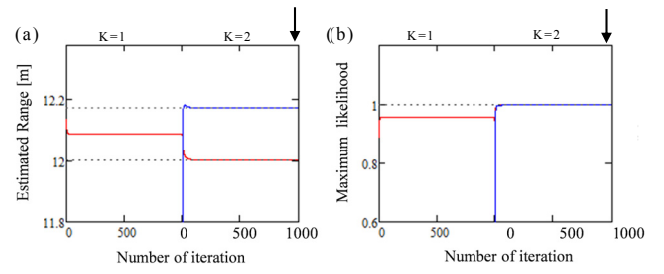


Fig. 3. An estimation result vs. number of iteration in case I ((a) Range estimation result, (b) Maximum value of likelihood function).

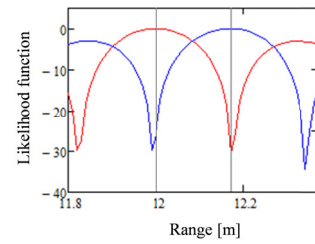


Fig. 4. The likelihood functions for targets 1 and 2 at the iteration number of 1000 in case I of the final phase.

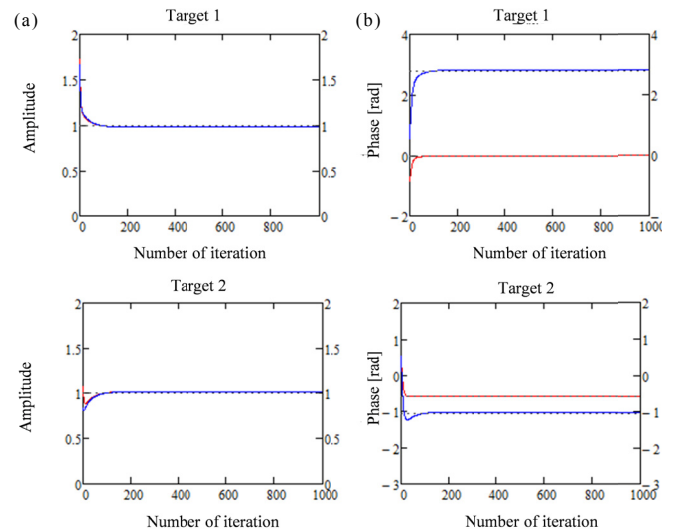


Fig. 5. An estimated  $\hat{\mathbf{a}}_{IF}$  vs. number of iteration in case I ((a) amplitude, (b) phase (freq.1: red, freq.2: blue))

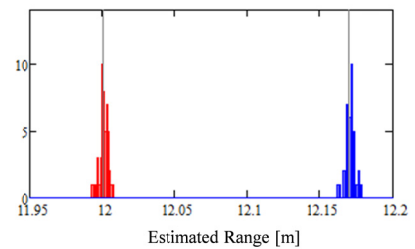


Fig. 6. Histogram of range estimation results of 50 trials for targets 1 and 2 in case I.

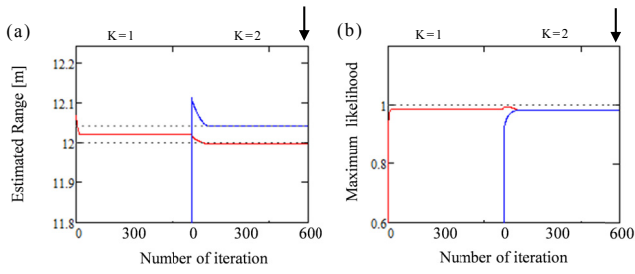


Fig. 7. An estimation result vs. number of iteration in case II ((a) Range estimation result, (b) Maximum value of likelihood function).

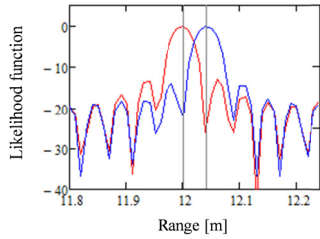


Fig. 8. The likelihood functions for targets 0 and 1 at the iteration number of 600 in case II.

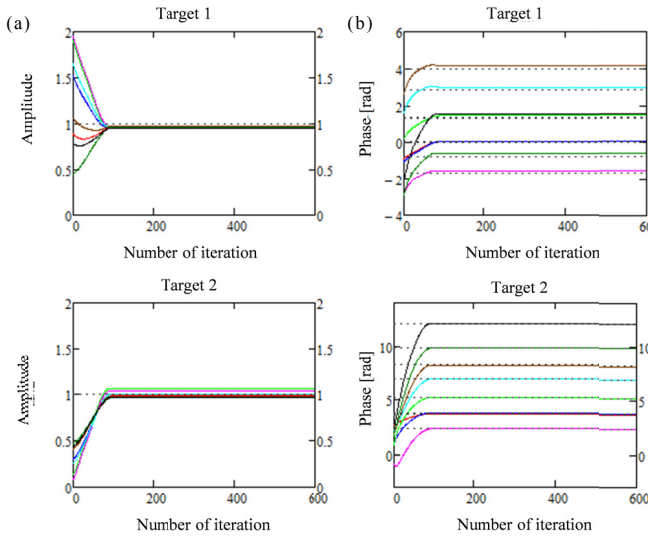


Fig. 9. An estimated  $\hat{\alpha}_{iF}$  vs. number of iteration in case II ((a) amplitude, (b) phase (freq.1: red, freq.2: blue, freq.3: light green, freq.4: magenta, freq.5: sky blue, freq.6: brown, freq.7: black, freq.8: dark green)).

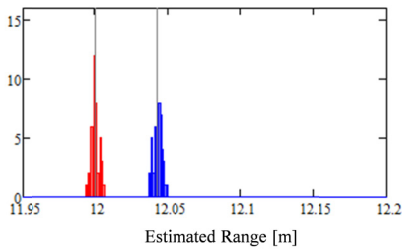


Fig. 10. Histogram of range estimation results of 50 trials for targets 1 and 2 in case II.

#### 4.1 Case II

Fig. 7 shows the range estimation result vs. number of iteration in depicted one case. From Fig.7, it is found that the estimated ranges of targets converge to the true value and the likelihood functions converge to 1. The range estimation results (12.000, 12.042) are obtained from the convergence values of target ranges. Fig. 8 shows the likelihood functions for targets 1 and 2 at the iteration number of 600 of the final phase of CCSM processing as indicated by arrows in Fig.7. The beam width of the likelihood functions are 0.042m that corresponds to the bandwidth of 3.5GHz. That indicated that the coherently combining in CCSM successfully works as in the case with case I. Fig. 9 shows the estimation result of  $\hat{\alpha}_{iF}$  vs. number of iteration of the final phase of CCSM processing. From Fig.9, it is also found that the all components, which are absolute value of amplitude for each target for each frequency band, of estimated  $\hat{\alpha}_{iF}$  converge to the true complex amplitudes in the case where many frequency bands are used. Fig. 10 shows the simulation results of 50 trials. The standard deviation of range estimations are also obtained as 0.003 for two targets.

#### 5. Conclusions

Authors have been developing a range estimation method with a high-resolution to expand the transmission bandwidth by coherent signal processing using measured radar data operated in separated bands. It is expected that this technique avoids the increase of a hardware load without the degradation of the detectable range, which generally become a problem associated with the expansion of the transmission bandwidth. In this paper, we explained the receiving signal model before synthetic bandwidth of MF radars operating in separated bands and described the iterative range estimation method by CCSM processing. We also show the simulation results using 2 and 8 separated frequency bands data where the two targets are separated with 0.170m and 0.042m equivalent to the range resolution of transmission bandwidth, respectively.

In future works, the simulation will be conducted in the situation where more than 3 targets are located. Then the CCSM processing will be also applied to the input data of

synthetic bandwidth of the stepped multiple frequency CPC of simulations and experiments.

## Acknowledgement

This research was supported by Strategic Information and Communications R&D Promotion Programme (SCOPE) of Ministry of Internal Affairs and Communications (MIC).

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