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An Analysis of 3-dimensional Location and Velocity Estimation Using Range and Doppler Measurements

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Abstract In a TOA (Time of Arrival) system such as the GPS, a target location can be estimated from several distance (range) measurements between a target location and reference points by the Taylor-series estimation. However, accuracy of estimated target location is very sensitive to target and reference points geometry. In this paper, we illustrate theoretically that we can always estimate 3-dimensional target location and velocity using several range and Doppler measurements when we can estimate 3-dimensional target location using the conventional TOA system with only range measurements. We also prove that estimated location accuracy becomes better when using additional Doppler measurements even in cases of poor target and reference points geometry or bad Doppler measurement accuracy.

Keyword TOA, GPS, location system, error analysis, range, Doppler

1. Introduction

In a TOA (Time of Arrival) system such as the GPS (Global Positioning System), a target location can be estimated from several range measurements between a target location and a reference point at known location as shown in Fig.1 by Taylor-Series estimation [1]~[5].

There are two approaches to obtain these range measurements. The first approach is to obtain the range measurements between a receiver at known location and the target that have a transmitter. The second approach is to obtain the range measurements between a transmitter at known location and the target that have a receiver such as the GPS [1]~[4].

In this paper, we assume that the clocks of all reference points are synchronized and there exists a clock offset between the transmitter clock and the receiver clock in a TOA system [1]~[4]. Then, it is necessary to obtain at least four range measurements to estimate 3-dimensional target location and the clock offset. Actually, we can estimate these four unknowns by the Taylor-series estimation [1]~[3].

Here, Taylor-series estimation (also Gauss-Newton algorithm) is an iterative method for solving non-linear least squares problems, starting with an initial estimate and converging the estimate.

By the way, target tracking is the processing of target measurements obtained by a sensor in order to maintain an estimate of its current and future state such as position and velocity assuming that noisy target measurements are obtained by a sensor, usually only position measurements [6]~[9].

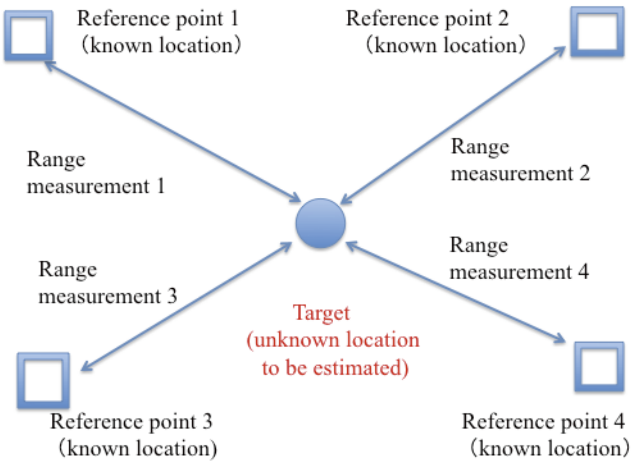

Fig.1 TOA

It was reported that the tracking performance using both position and velocity measurements could be better than that of tracking filter using only position measurements [9].

Therefore, accuracy of GPS tracking could be improved when we estimate position and velocity by using both range and Doppler measurements.

However, accuracy of position measurements does not have deterioration by obtaining velocity measurements is a prerequisite for improving tracking performance.

Furthermore, accuracy of estimated target location is very sensitive to target and reference points geometry in the TOA.

In this paper, we make it clear whether we can estimate 3-dimensional target location and velocity using several range and Doppler measurements when we can estimate 3-dimensional target location using the conventional TOA system. We also make it clear whether estimated location accuracy becomes better when using additional Doppler measurements even in cases of poor target and reference points geometry or bad Doppler measurement accuracy.

2. Location and velocity estimation

In this chapter, we assume that we can obtain n pairs of range and Doppler measurements from n reference points. Let D^T be the transpose of a matrix D.

2.1. Range measurement model

Let $\underline{B}_i$ be a known location vector of an $i(i=1,2,\cdots,n)$ th reference point in 3-dimensional Cartesian coordinates given by:

$$\underline{B}_i = (x_i, y_i, z_i)^T \quad (1)$$

Let $\underline{L}$ be an unknown location vector of the target in 3-dimensional Cartesian coordinates given by:

$$\underline{L} = (x, y, z)^T \quad (2)$$

Then, the true range (distance) R_i between the target location and the ith reference point is defined as:

$$R_i = f_i(x, y, z) \quad (3)$$

where

$$f_i(x,y,z) = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \quad (4)$$

Therefore, the range measurement R_{io} between a target location and the i th reference point can be obtained as:

$$R_{io} = R_i + S + v_i \quad (5)$$

where $R_i + S$ is the noiseless pseudorange, S is the range bias caused by the clock offset between the transmitter clock and the receiver clock, v_i is the random range measurement noise.

Then, we obtain the following property using the total differential (Taylor-series linearization).

(Property1) Let x_0, y_0, z_0 be an initial (nominal) estimate of the target location. Then the following result holds:

$$\Delta R_{io} = \begin{pmatrix} \alpha_i & \beta_i & \gamma_i & 1 \end{pmatrix} \underline{a}_l + v_i \quad (6)$$

where

$$\Delta R_{io} = R_{io} - f_i(x_0, y_0, z_0) \quad (7)$$

$$\underline{a}_l = (x - x_0, y - y_0, z - z_0, S)^T \quad (8)$$

$$\alpha_i = -\frac{x_i - x_0}{f_i(x_0, y_0, z_0)}, \beta_i = -\frac{y_i - y_0}{f_i(x_0, y_0, z_0)}, \gamma_i = -\frac{z_i - z_0}{f_i(x_0, y_0, z_0)} \quad (9)$$

Here, $(\alpha_i, \beta_i, \gamma_i)$ is the unit vector from the reference point i to the initial target location $(x_0, y_0, z_0)^T$ as shown in Fig.2.

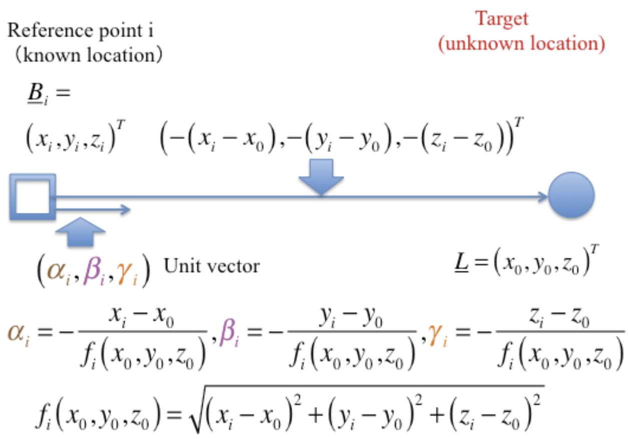

Fig.2 Linear approximation coefficients

2.2. Doppler measurement model

Let $\dot{\underline{B}}_i$ be a known velocity vector of an $i(i=1,2,\cdots,n)$ th reference point in 3-dimensional Cartesian coordinates given by:

$$\dot{\underline{B}}_i = (\dot{x}_i, \dot{y}_i, \dot{z}_i)^T \quad (10)$$

Let $\dot{\underline{L}}$ be an unknown vector vector of the target in 3-dimensional Cartesian coordinates given by:

$$\dot{\underline{L}} = (\dot{x}, \dot{y}, \dot{z})^T \quad (11)$$

Then, the true Doppler $\dot{R}_i$ between the target location and the ith reference point is defined as:

$$\dot{R}_i = g_i(x, y, z, \dot{x}, \dot{y}, \dot{z}) \quad (12)$$

where

$$g_i(x,y,z,\dot{x},\dot{y},\dot{z}) = \frac{(x_i-x)(\dot{x}_i-\dot{x})+(y_i-y)(\dot{y}_i-\dot{y})+(z_i-z)(\dot{z}_i-\dot{z})}{f_i(x,y,z)} \quad (13)$$

Therefore, the Doppler measurement $\dot{R}_{io}$ between a target location and the i th reference point can be obtained as:

$$\dot{R}_{io} = \dot{R}_i + \dot{v}_i \quad (14)$$

where $\dot{v}_i$ is the random Doppler measurement noise.

Then, we obtain the following property using the total differential (Taylor-series linearization).

(Property2) Let $\dot{x}_0, \dot{y}_0, \dot{z}_0$ be an initial (nominal) estimate of the target velocity. Then the following result holds:

$$\Delta\dot{R}_{io} = \begin{pmatrix} \alpha_{il} & \beta_{il} & \gamma_{il} & 0 & \alpha_i & \beta_i & \gamma_i \end{pmatrix} \underline{a} + \dot{v}_i \quad (15)$$

where

$$\Delta\dot{R}_{io} = \dot{R}_{io} - g_i(x_0,y_0,z_0,\dot{x}_0,\dot{y}_0,\dot{z}_0) \quad (16)$$

$$\underline{a} = (x-x_0, y-y_0, z-z_0, S, \dot{x}-\dot{x}_0, \dot{y}-\dot{y}_0, \dot{z}-\dot{z}_0)^T \quad (17)$$

$$\begin{aligned} \alpha_{il} &= -\frac{(\dot{x}_i-\dot{x}_0) f_i(x_0,y_0,z_0) - g_i(x_0,y_0,z_0,\dot{x}_0,\dot{y}_0,\dot{z}_0)(x_i-x_0)}{f_i(x_0,y_0,z_0)^2} \\ \beta_{il} &= -\frac{(\dot{y}_i-\dot{y}_0) f_i(x_0,y_0,z_0) - (y_i-y_0) g_i(x_0,y_0,z_0,\dot{x}_0,\dot{y}_0,\dot{z}_0)}{f_i(x_0,y_0,z_0)^2} \\ \gamma_{il} &= -\frac{(\dot{z}_i-\dot{z}_0) f_i(x_0,y_0,z_0) - (z_i-z_0) g_i(x_0,y_0,z_0,\dot{x}_0,\dot{y}_0,\dot{z}_0)}{f_i(x_0,y_0,z_0)^2} \end{aligned} \quad (18)$$

2.3. Linear model of measurements

The following linear model can be obtained from Eqs.(6) and (15) when n reference points are used:

$$\underline{b} = A\underline{a} + \underline{v} \quad (19)$$

where

$$\underline{b} = (\Delta R_{1o}, \Delta R_{2o}, \cdots, \Delta R_{no}, \Delta\dot{R}_{1o}, \Delta\dot{R}_{2o}, \cdots, \Delta\dot{R}_{no})^T \quad (20)$$

$$A = \begin{pmatrix} \alpha_1 & \beta_1 & \gamma_1 & 1 & 0 & 0 & 0 \\ \alpha_2 & \beta_2 & \gamma_2 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_n & \beta_n & \gamma_n & 1 & 0 & 0 & 0 \\ \alpha_{1l} & \beta_{1l} & \gamma_{1l} & 0 & \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_{2l} & \beta_{2l} & \gamma_{2l} & 0 & \alpha_2 & \beta_2 & \gamma_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{nl} & \beta_{nl} & \gamma_{nl} & 0 & \alpha_n & \beta_n & \gamma_n \end{pmatrix} \quad (21)$$

$$\underline{v}_l = (v_1, v_2, \cdots, v_n)^T \quad (22)$$

$$\underline{v}_d = (\dot{v}_1, \dot{v}_2, \cdots, \dot{v}_n)^T \quad (23)$$

$$\underline{v} = (\underline{v}_l^T, \underline{v}_d^T)^T \quad (24)$$

and we call A a placement matrix.

By the way, we assume that measurement noises are zero mean and uncorrelated among the sensors as follows:

$$E[\underline{v}] = \underline{0} \quad (25)$$

$$V = E[\underline{v}\underline{v}^T] = \begin{pmatrix} V_l & V_{ld} \\ V_{ld} & V_d \end{pmatrix} \quad (26)$$

$$V_l = E[\underline{v}_l \underline{v}_l^T] = diag\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_n^2\} \quad (27)$$

$$V_d = E[\underline{v}_d \underline{v}_d^T] = diag\{\sigma_{1d}^2, \sigma_{2d}^2, \cdots, \sigma_{nd}^2\} \quad (28)$$

$$V_{ld} = E[\underline{v}_l \underline{v}_d^T] = diag\{\rho_1\sigma_1\sigma_{1d}, \rho_2\sigma_2\sigma_{2d}, \cdots, \rho_n\sigma_n\sigma_{nd}\} \quad (29)$$

Here, $E[\ ]$ indicates the mean, $diag\{a_1, a_2, \cdots, a_n\}$ indicates the diagonal matrix whose elements are $a_1, a_2, \cdots, a_n$.

To simplify a description, we define the following equations using Eqs.(9) and (18):

$$\underline{\omega}(i) = \begin{pmatrix} \alpha_i & \beta_i & \gamma_i \end{pmatrix} \quad (30)$$

$$\underline{\delta}(i) = \begin{pmatrix} \underline{\omega}(i) & 1 \end{pmatrix} \quad (31)$$

$$\underline{\kappa}(i) = \begin{pmatrix} \alpha_{il} & \beta_{il} & \gamma_{il} & 0 \end{pmatrix} \quad (32)$$

$$A_l = \begin{pmatrix} \underline{\delta}(1)^T & \cdots & \underline{\delta}(n)^T \end{pmatrix}^T \quad (33)$$

$$A_{ld} = \begin{pmatrix} \underline{\kappa}(1)^T & \cdots & \underline{\kappa}(n)^T \end{pmatrix}^T \quad (34)$$

$$A_d = \begin{pmatrix} \underline{\omega}(1)^T & \cdots & \underline{\omega}(n)^T \end{pmatrix}^T \quad (35)$$

Then we can obtain the following equation from Eq. (21):

$$A = \begin{pmatrix} A_l & O_{n,3} \\ A_{ld} & A_d \end{pmatrix} \quad (36)$$

Here, $O_{m,n}$ is an $m \times n$ zero matrix.

2.4. Weighed least-squares method

The following property shows that we can estimate the target location and velocity using the weighed least-squares method [10], [11]. Namely, we minimize the following equation to obtain an optimal estimate $\hat{\underline{a}}$ for the location vector, velocity vector and the range bias:

$$J = (\underline{b} - A\hat{\underline{a}})^T V^{-1} (\underline{b} - A\hat{\underline{a}}) \quad (37)$$

Here, D^{-1} is an inverse of a matrix D.

Then, we obtain the following property.

(Property3) Let $A^T V^{-1} A$ be invertible. Then the following result holds:

$$\hat{\underline{a}} = \left(A^T V^{-1} A\right)^{-1} A^T V^{-1} \underline{b} \quad (38)$$

The following property shows that $\hat{\underline{a}}$ is the unbiased estimate and its estimated covariance matrix.

(Property4) The following result holds:

$$E[\hat{\underline{a}}] = \underline{a} \quad (39)$$

$$E\left[(\hat{\underline{a}} - \underline{a})(\hat{\underline{a}} - \underline{a})^T\right] = \left(A^T V^{-1} A\right)^{-1} \quad (40)$$

2.5. Observability

The following property shows that if the rank of matrix A_l is 4, then the rank of matrix A_d is 3.

(Property5) Let $4 \le n$. Then, we can find 3 independent vectors among $\underline{\omega}(i), \underline{\omega}(j), \underline{\omega}(k), \underline{\omega}(l)$ in the matrix A_d if we can find 4 independent $\underline{\delta}(i), \underline{\delta}(j), \underline{\delta}(k), \underline{\delta}(l)$ $(i < j < k < l)$ in the matrix A_l and the rank of matrix A is 7.

The following property shows that $A^T V^{-1} A$ has inverse if the rank of matrix A_l is 4 and $V > 0$. Here, $D > 0$ indicates that a matrix D is positive definite, and $D \ge 0$ indicates that a matrix D is positive semi-definite.

(Property6) Let $4 \le n$ and Q be a $2n \times 2n$ positive-definite matrix. Then, $A^T Q A$ is invertible if we can find 4 independent $\underline{\delta}(i), \underline{\delta}(j), \underline{\delta}(k), \underline{\delta}(l)$ $(i < j < k < l)$ in the matrix A_l.

The following property shows that we can determine whether $A^T V^{-1} A$ has inverse only using the unit vector $\underline{\omega}(i)$.

(Property7) Let $4 \le n$, Q be a $2n \times 2n$ positive-definite matrix and k be one of $1, \cdots, n$. Then, $A^T Q A$ is invertible if we can find 3 independent $\underline{\omega}(j) - \underline{\omega}(k)$ $(j = 1, \cdots, n, j \ne k)$.

2.6. Measurement noise covariance matrix

The following property shows that V is the non-singular matrix if the absolute value of correlation coefficient ρ_i between range and Doppler measurements is less than 1.

(Property8) Let $\sigma_i > 0, \sigma_{id} > 0, |\rho_i| < 1$ $(i = 1, \cdots, n)$. Then, the following result holds:

$$V > 0 \quad (41)$$

3. Comparison of estimated location accuracy

In this chapter, we compare the estimated location accuracy between the conventional TOA only using range measurements and the TOA using additional Doppler measurements theoretically.

3.1. Conventional TOA

The following linear model of range can be obtained from Eqs.(6), (30), (31), (33), (8) and (22) when n reference points are used:

$$\underline{b}_l = A_l \underline{a}_l + \underline{v}_l \quad (42)$$

where

$$\underline{b}_l = \left(\Delta R_{1o}, \Delta R_{2o}, \cdots, \Delta R_{no}\right)^T \quad (43)$$

The following property shows that we can estimate the target location and range bias using the weighed least-squares method without Doppler measurements [10], [11]. Namely, we minimize the following equation to obtain an optimal estimate $\hat{\underline{a}}_l$ for the location vector and the range bias:

$$J_l = (\underline{b}_l - A_l \hat{\underline{a}}_l)^T V_l^{-1} (\underline{b}_l - A_l \hat{\underline{a}}_l) \quad (44)$$

Then, we obtain the following property.

(Property9) Let $A_l^T V_l^{-1} A_l$ be invertible. Then the following result holds:

$$\hat{\underline{a}}_l = \left(A_l^T V_l^{-1} A_l\right)^{-1} A_l^T V_l^{-1} \underline{b}_l \quad (45)$$

By the way, $A_l^T V_l^{-1} A_l$ is invertible if and only if we can find 3 independent $\underline{\omega}(j) - \underline{\omega}(k)$ $(j = 1, \cdots, n, j \ne k)$ [3].

The following property shows that $\hat{\underline{a}}_l$ is the unbiased estimate and its estimated covariance matrix.

(Property10) The following result holds:

$$E[\hat{\underline{a}}_l] = \underline{a}_l \quad (46)$$

$$E\left[(\hat{\underline{a}}_l - \underline{a}_l)(\hat{\underline{a}}_l - \underline{a}_l)^T\right] = \left(A_l^T V_l^{-1} A_l\right)^{-1} > 0 \quad (47)$$

3.2. Estimation with Doppler measurements

The following equation, Eqs.(8) and (17) show the estimated location and range bias in the TOA with Doppler measurements.

$$\underline{\hat{a}}_D = N\underline{\hat{a}} \quad (48)$$

where

$$N = \begin{pmatrix} I_4 & O_{4,3} \end{pmatrix} \quad (49)$$

The following property shows that $\underline{\hat{a}}_D$ is the unbiased estimate and its estimated covariance matrix.

(Property11) The following result holds:

$$E\left[\underline{\hat{a}}_D\right] = \underline{a}_l \quad (50)$$

$$E\left[\left(\underline{\hat{a}}_D - \underline{a}_l\right)\left(\underline{\hat{a}}_D - \underline{a}_l\right)^T\right] = N\left(A^T V^{-1} A\right)^{-1} N^T \quad (51)$$

The property11 can be proved analytically using the property4.

3.3. Comparison

It is necessary to evaluate $\underline{\hat{a}}_l$ (without Doppler measurements) and $\underline{\hat{a}}_D$ (with Doppler measurements) to compare the estimated location and range bias accuracy.

Eqs.(46) and (50) show that both $\underline{\hat{a}}_l$ and $\underline{\hat{a}}_D$ are the unbiased estimate.

The following property shows that estimated location and range bias accuracy of the TOA with additional Doppler measurements is better than that of the conventional TOA only using range measurements because Eqs.(47) and (51) hold.

(Property12) Let $4 \le n$, k be one of $1,\cdots,n$ and $\sigma_i > 0, \sigma_{id} > 0, |\rho_i| < 1 \ (i = 1,\cdots,n)$. Then, the following result holds if we can find 3 independent $\underline{\omega}(j) - \underline{\omega}(k)$ $(j = 1,\cdots,n, j \ne k)$.

$$N\left(A^T V^{-1} A\right)^{-1} N^T \le \left[A_l^T V_l^{-1} A_l\right]^{-1} \quad (52)$$

The property12 can be proved analytically using the matrix theory.

4. Consideration

4.1. Observability

When we can estimate 3-dimensional target location and range bias using the conventional TOA, we can find find 3 independent $\underline{\omega}(j) - \underline{\omega}(k)$ $(j = 1,\cdots,n, j \ne k)$ in the matrix A_l and then we can also estimate 3-dimensional target location, velocity and range bias using additional Doppler measurements (see property7).

4.2. Numerical results

Numerical calculations are conducted in order to present an example where there exists the difference between the TOA with additional Doppler measurements and the conventional TOA only using range measurements in terms of the estimated location error variance.

We assume that a target that has a transmitter and 3 receivers are on the plane for simplicity.

We assume that the target and receivers are located at

$$\underline{B}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \underline{B}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \underline{B}_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underline{L} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (53)$$

as shown in Fig.3.

We assume that the velocity vectors are as follows.

$$\underline{\dot{B}}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underline{\dot{B}}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underline{\dot{B}}_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underline{\dot{L}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (54)$$

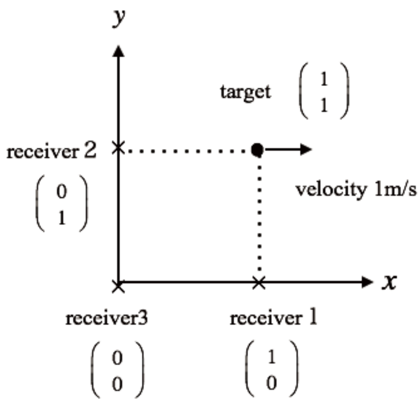

Fig.3 Target and receivers geometry

We also assume that $\left(x_0, y_0\right)^T = \underline{L}$, $\left(\dot{x}_0, \dot{y}_0\right)^T = \underline{\dot{L}}$, $S > 0$, and $\sigma_i^2 = 0.01, \sigma_{id}^2 = a^2, \rho_i = 0 \left(i = 1,2,3\right)$.

Then, we can obtain the following results from Eqs.(33)~(36).

$$A = \begin{pmatrix} A_l & O_{3,2} \\ A_{ld} & A_3 \end{pmatrix} \quad (55)$$

$$A_l = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{pmatrix} \quad (56)$$

$$A_{ld} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \end{pmatrix} \quad (57)$$

$$A_d = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (58)$$

Let estimated location error variances of the TOA with additional Doppler measurements and the conventional TOA be P_{ld}, P_l, respectively. Here, variances of estimated location error of x coordinate are the same as that of y coordinate. Then, we can obtain the following results theoretically.

$$P_l = 0.01\left(3\sqrt{2}+5\right) \tag{59}$$

$$P_{ld} = \frac{0.01}{16a^2+0.01}\cdot\frac{24a^2+0.01}{2} + \frac{0.01}{8\left(3\sqrt{2}-4\right)a^2+3\sqrt{2}\cdot 0.01}\cdot\frac{32\left(3\sqrt{2}+4\right)a^4}{16a^2+0.01} \tag{60}$$

Then we obtain the following results from Eqs.(59) and (60).

$$P_l > P_{ld} \tag{61}$$

$$P_{ld} \to P_l \quad \left(a^2 \to \infty\right) \tag{62}$$

$$P_{ld} \to 0.01/2 \quad \left(a^2 \to 0\right) \tag{63}$$

Fig.4 shows the ratio of estimated location error variance (P_{ld}/P_l) as the function of variances of Doppler measurements a^2.

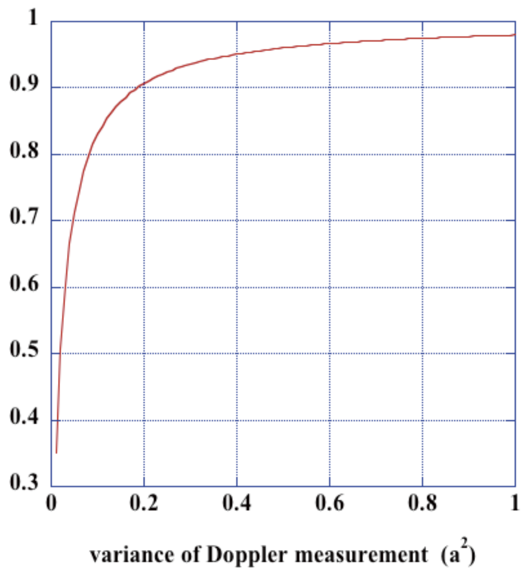

Fig.4 Ratio of estimated location error variance

Eq.(61) shows that there exists an example where estimated location accuracy of the TOA with additional Doppler measurements is not equal to that of the conventional TOA only using range measurements.

Eq.(62) shows that there is no difference between the TOA with additional Doppler measurement and the conventional TOA only using range measurements in terms of estimated location accuracy when the variance of Doppler measurement is very large .

Eq.(63) shows that estimated location error variance of the TOA with additional Doppler measurements converges to half of range measurement error variance when the variance of Doppler measurement are very small .

5. Conclusions

In this paper, we illustrated that when we can estimate 3-dimensional target location using the conventional TOA system only using range measurements, we can also estimate 3-dimensional target location and velocity using several range and Doppler measurements. We also showed that estimated location accuracy becomes better when using additional Doppler measurements even in cases of poor target and reference points geometry or bad Doppler measurement accuracy theoretically.

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